PROOF WITHOUT WORDS

The Taylor Polynomials of sin θ

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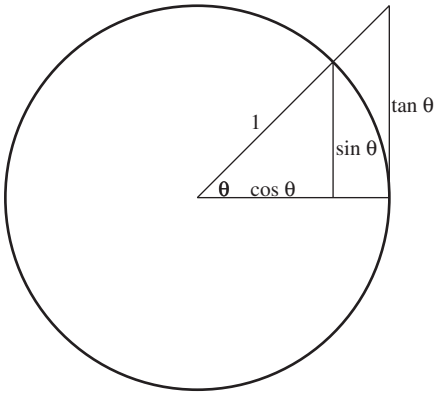
I.



$$\frac{1}{2}\sin\theta\cos\theta < \frac{1}{2}\theta < \frac{1}{2}\,\frac{\sin\theta}{\cos\theta}$$

$$\cos\theta < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$$

$$\downarrow \qquad\qquad \downarrow$$

$$1 \qquad\qquad 1$$

$$\sin\theta \approx \theta \qquad (\theta \approx 0)$$

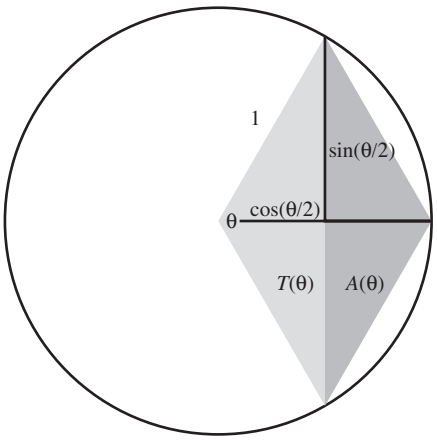
II(a).

$$T(\theta) = \frac{1}{2}\left[2\sin\left(\frac{\theta}{2}\right)\right]\left[\cos\left(\frac{\theta}{2}\right)\right] = \frac{\sin\theta}{2}$$

$$A(\theta) = \frac{1}{2}\left[2\sin\left(\frac{\theta}{2}\right)\right]\left[1-\cos\left(\frac{\theta}{2}\right)\right] = \sin\left(\frac{\theta}{2}\right)\left[2\sin^2\left(\frac{\theta}{4}\right)\right]$$

$$= 2\sin\left(\frac{\theta}{2}\right)\sin^2\left(\frac{\theta}{4}\right)$$

$$\approx 2\left(\frac{\theta}{2}\right)\left(\frac{\theta}{4}\right)^2 = \frac{\theta^3}{16}$$



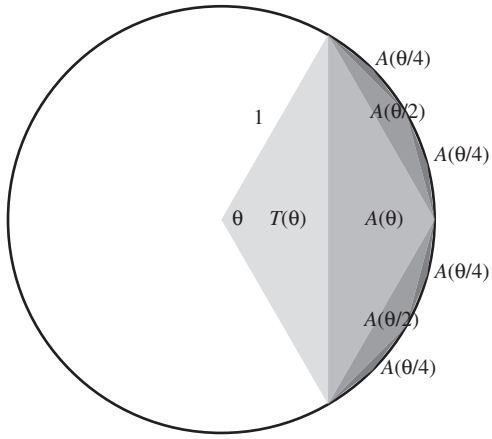
II(b).

$$\frac{\theta}{2} = T(\theta) + A(\theta) + 2A(\theta/2) + 4A(\theta/4) + \cdots$$

$$\frac{\theta}{2} \approx \frac{\sin\theta}{2} + \frac{\theta^3}{16} + 2\left[\frac{(\theta/2)^3}{16}\right] + 4\left[\frac{(\theta/4)^3}{16}\right] + \cdots$$

$$\frac{\theta}{2} \approx \frac{\sin\theta}{2} + \frac{\theta^3}{12}$$

$$\theta - \frac{\theta^3}{6} \approx \sin\theta$$



III(a).

$$A(\theta) = 2\sin\left(\frac{\theta}{2}\right)\sin^2\left(\frac{\theta}{4}\right) \approx 2\left[\frac{\theta}{2} - \frac{(\theta/2)^3}{6}\right]\left[\frac{\theta}{4} - \frac{(\theta/4)^3}{6}\right]^2 \approx \frac{\theta^3}{16} - \frac{\theta^5}{256}$$

III(b).

$$\frac{\theta}{2} = T(\theta) + A(\theta) + 2A(\theta/2) + 4A(\theta/4) + \cdots$$

$$\frac{\theta}{2} \approx \frac{\sin\theta}{2} + \left[\frac{\theta^3}{16} - \frac{\theta^5}{256}\right] + 2\left[\frac{(\theta/2)^3}{16} - \frac{(\theta/2)^5}{256}\right] + 4\left[\frac{(\theta/4)^3}{16} - \frac{(\theta/4)^5}{256}\right] + \cdots$$

$$\frac{\theta}{2} \approx \frac{\sin\theta}{2} + \frac{\theta^3}{12} - \frac{\theta^5}{240}$$

$$\theta - \frac{\theta^3}{6} + \frac{\theta^5}{120} \approx \sin\theta$$

ETC.